Comparison of the tangent linear properties of tracer transport schemes applied to geophysical problems.

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INTRODUCTION

A number of geophysical applications require the use of the linearized version of the full model. One such example is in numerical weather prediction, where the tangent linear and adjoint versions of the atmospheric model are required for the 4DVAR inverse problem.

The part of the model that represents the resolved scale processes of the atmosphere is known as the dynamical core. Advection, or transport, is performed by the dynamical core. It is a central process in many geophysical applications and is a process that often has a quasi-linear underlying behavior. However, over the decades since the advent of numerical modelling, significant effort has gone into developing many flavors of high-order, shape preserving, non-oscillatory, positive definite advection schemes. These schemes are excellent in terms of transporting the quantities of interest in the dynamical core, but they introduce nonlinearity through the use of nonlinear limiters.

The linearity of the transport schemes used in Goddard Earth Observing System version 5 (GEOS-5), as well as a number of other schemes, is analyzed using a simple 1D setup. The linearized version of GEOS-5 is then tested using a linear third order scheme in the tangent linear version.

1D CASE STUDY

The one dimensional advection equation is given by,

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} = 0, \tag{1}$$

where $q(x,t)$ is the tracer being advected, u is the wind and t is time. The horizontal domain is periodic on $x \in (0,1]$. The number of grid points is $N = 64$ and the grid spacing is $\Delta x = 1/N$. The velocity is constant and set to $u = 1$ and the Courant number is chosen as 0.1 to ensure stability, giving a time step of $\Delta t = 1/640$.

Three initial profiles are considered. A step function,

$$q_j = \begin{cases} 1, & \text{if } 0.25 < x_j < 0.75 \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

a sine wave,

$$q_j = 0.5(1 + \sin(2\pi x_j)) \tag{3}$$

and a point function.

$$q_j = \begin{cases} 1, & \text{if } j = N/2 \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

The profiles are advected once around the domain using the schemes outlined in the table.

Scheme Type	Nonlinear limiter	Acronym
1st order FD	None	
2nd order FD	None	
3rd order FD	None	
PPM	None	
PPM	Colella and Woodward (1984)	CW
PPM	Colella and Sekora (2008)	CS
PPM	CW + Lin (2004)	CWL
3rd order FD	Leonard (1991) (universal)	UL
SLICE	None	
SLICE	Bermejo and Staniforth (1992)	BES

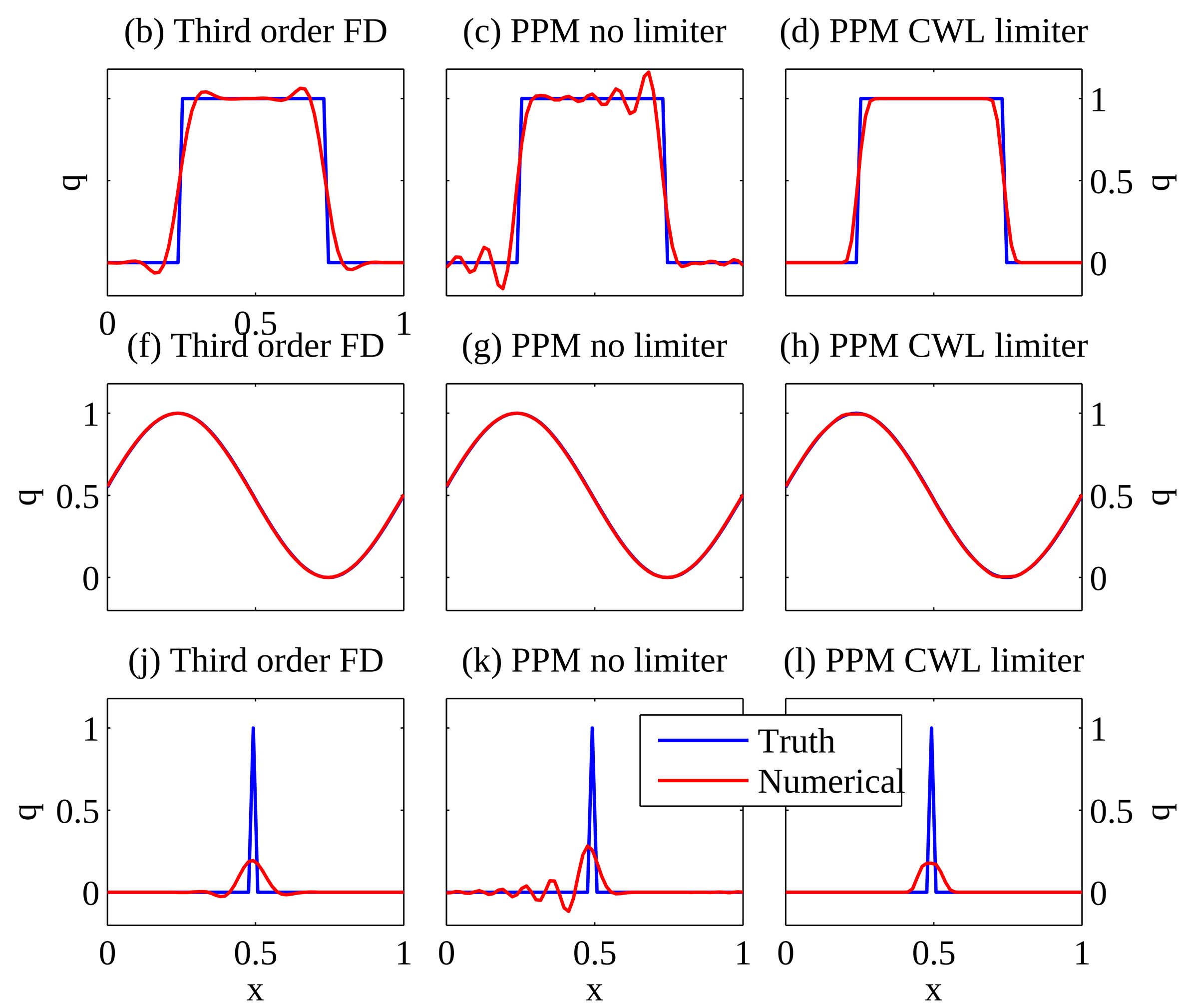

FIGURE 1: **This figure shows the advection of the three initial profiles using a third order scheme, an unlimited PPM scheme and the PPM scheme with CWL limiter that is used in GEOS-5. The limited PPM scheme prevents oscillations and negative values and maintains the shape most accurately.**

Various infinitesimal perturbations δq are applied to the initial conditions. Linearity is measured by comparing the nonlinear perturbation trajectory $\mathbf{m}(\mathbf{q}+\delta\mathbf{q}) - \mathbf{m}(\mathbf{q})$ with the linear perturbation trajectory $\mathbf{M}\delta\mathbf{q}$. The behavior of individual solutions can be ascertained by seeking wavelike solutions $\mathbf{q}' \sim \exp(\lambda t)$.

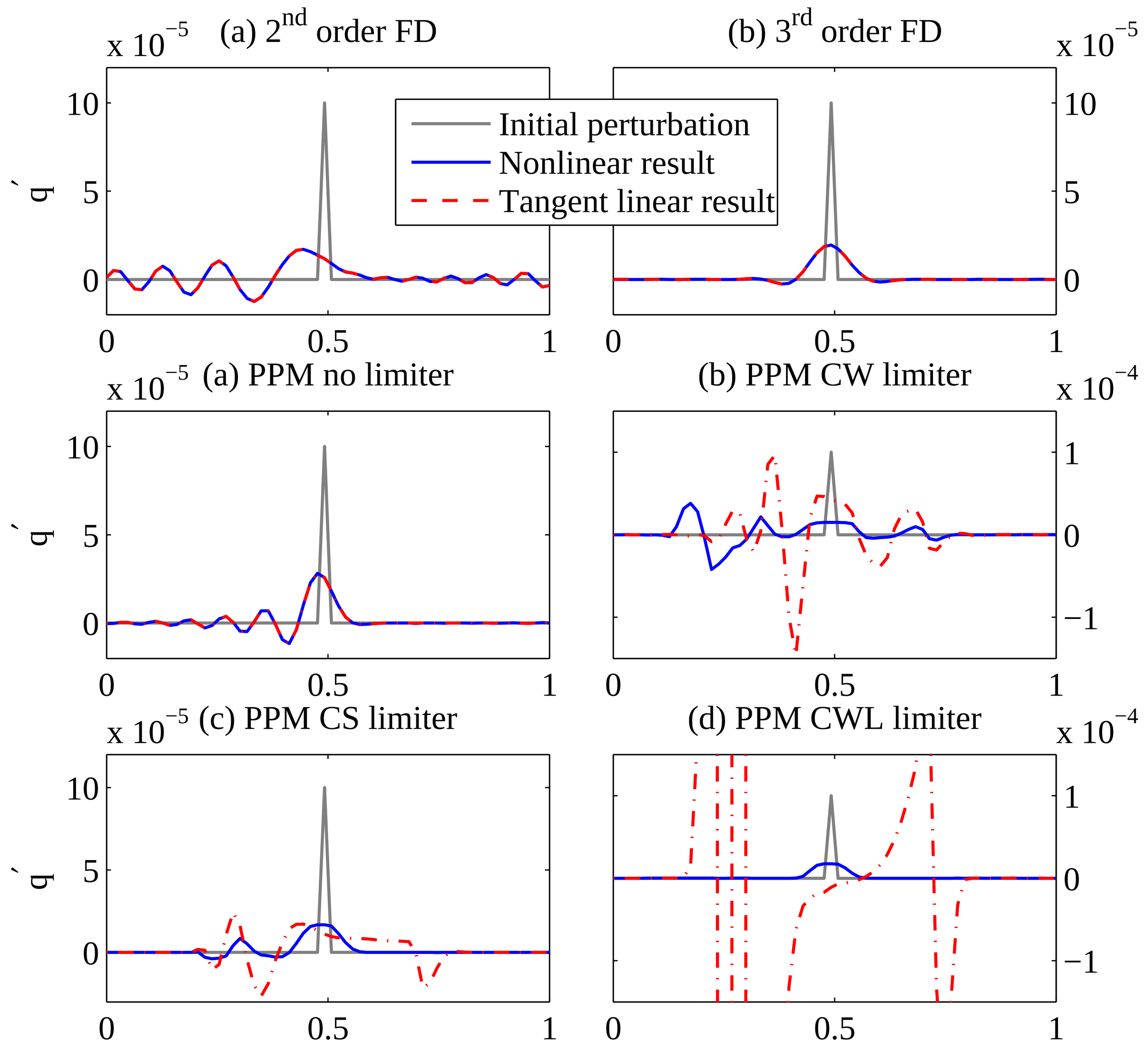

FIGURE 2: **This figure shows the advection of a perturbation applied to the step function. The schemes used are second order, third order, unlimited PPM and three limited PPM schemes. For linear schemes the linear and nonlinear perturbation trajectories are equivalent (red curves equal blue curves). The nonlinear schemes have large oscillations.**

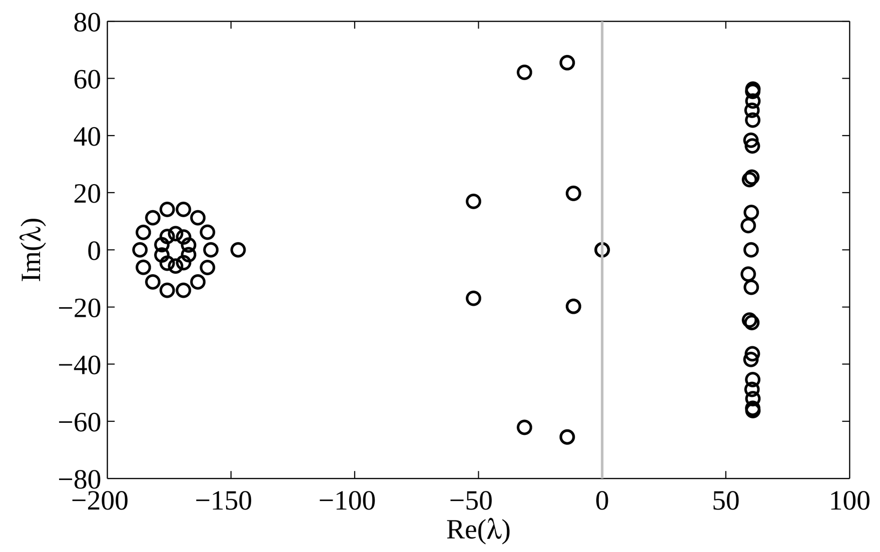

FIGURE 3: **This figure shows the complex-plane scatter plot of the eigenvalue spectrum for the CWL limited PPM scheme. Many solutions with large amplitude growth are present, leading to the development of spuriously large perturbations. This many growing modes are seen at most time steps.**

The 1D case study reveals that:

- If the perturbation has the same shape as the underlying function all schemes perform linearly.
- If the underlying profile is smooth then the nonlinear limiters are inactive, and the performance of the nonlinear schemes is similar to the linear part of the scheme.
- If the underlying profile contains discontinuities all of the nonlinear schemes have problems when linearized, e.g. PPM and UL schemes have growing modes.

GEOS-5 LINEARIZED MODEL

A simplified configuration of the linearized version of GEOS-5 is used in order to compare the behavior of the linear third order scheme with the default PPM with CWL limiter for realistic applications. A 24 hour long integration of the nonlinear model is initialized from realistic conditions. A second integration, and nonlinear perturbation trajectory, is obtained by adding an analysis increment to the initial conditions. That same analysis increment is propagated using the tangent linear model and compared with the nonlinear perturbation trajectory after 24 hours.

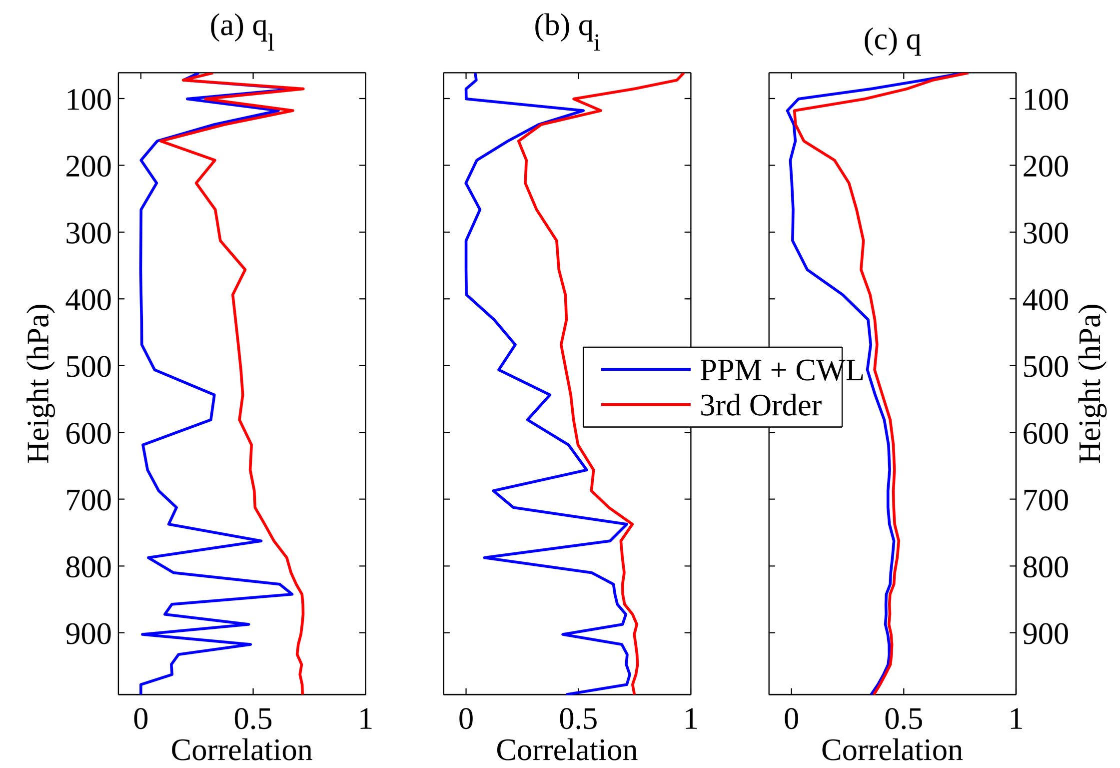

FIGURE 4: **Correlations for cloud liquid water, cloud liquid ice and specific humidity after 24 hours.**

When using the PPM scheme with CWL limiter perturbations can grow very large. Problems occur where the wind speeds are low. In regions where large perturbation growth does not occur, the errors for PPM CWL are generally larger than using the linear third order scheme.

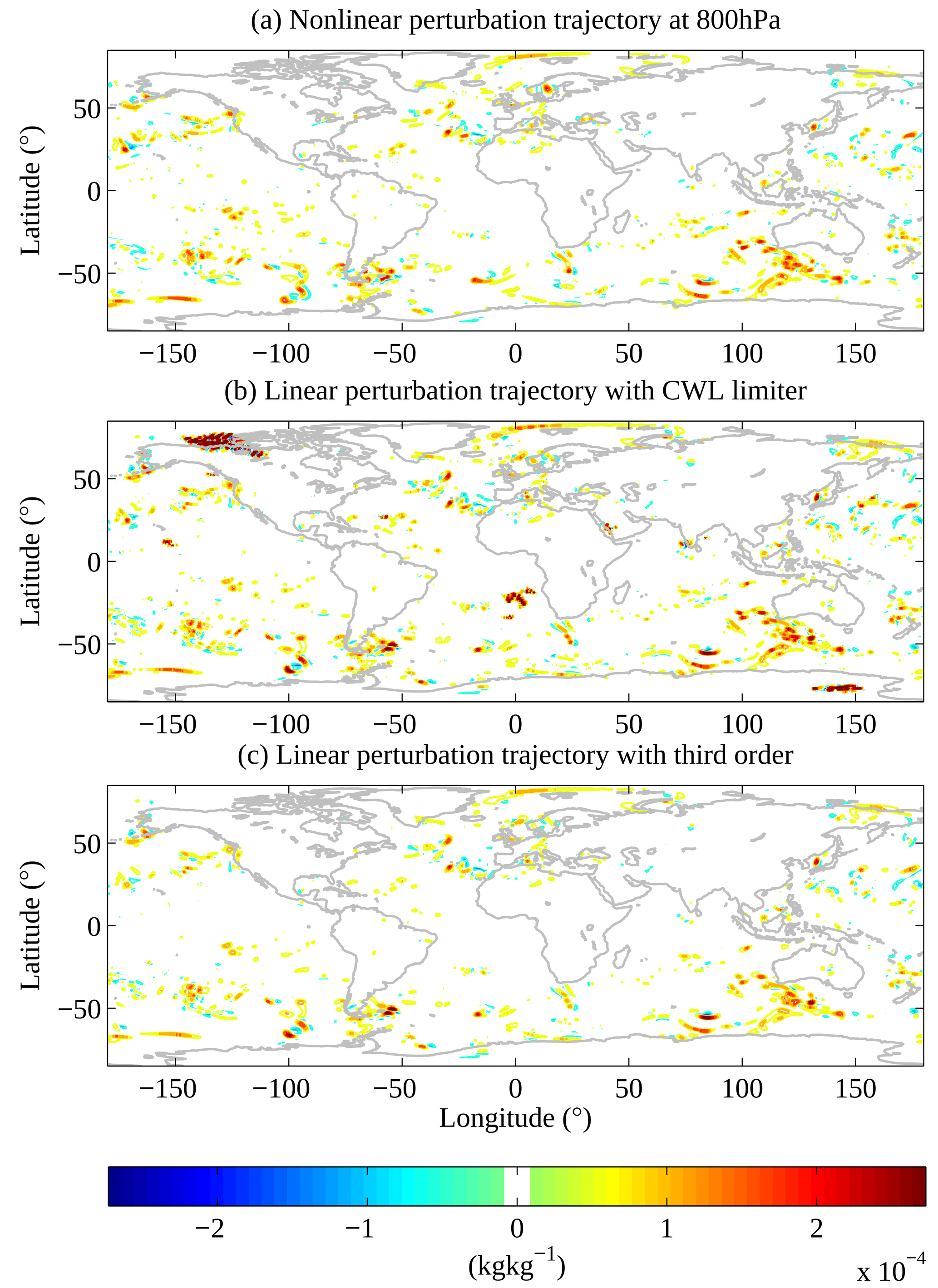

FIGURE 5: **This figure shows the nonlinear perturbations trajectory and the linear perturbation trajectory at 800hPa when using the PPM scheme with CWL limiter and the third order finite difference scheme. With the third order scheme no spurious perturbation growth is seen.**

When switching to the third order scheme the biggest improvement is seen for the most discontinuous fields, which are the clouds. Specific humidity and ozone also benefit significantly.

CONCLUSIONS

Motivated by finding a suitable advection scheme to use in the linearized version of GEOS-5, the linearity of a selection of common tracer transport schemes has been tested. All of the nonlinear schemes tested had some degree of problem. The PPM and UL schemes support rapidly growing solutions which cause unrealistic perturbation growth. It is argued that a third order linear scheme should be used in the linearized version of GEOS-5: it is linear, performs advection similarly to limited schemes, does not support large perturbation growth, does not produce significant negative values, has limited oscillations and is relatively inexpensive.

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